Reply to comments on `Comparison between two schemes for determining stable ground states in quantum dots'

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REPLY TO COMMENT

Reply to comments on 'Comparison between two schemes for determining stable ground states in quantum dots'

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Abstract. Kawamura *et al* have used the exact diagonalization results for quantum dots containing $N = 7$ and $N = 10$ electrons to review the predictions of cusp structure in the interaction energy given by two recently proposed schemes. Here we revisit the issues raised by them.

In the preceding paper Kawamura *et al* [1] have reviewed two schemes for predicting ground states of quantum dots in the light of exact diagonalization results for quantum dots in a strong magnetic field and containing seven and 10 electrons ($N = 7$, 10). One of the schemes was proposed by us [2] while the other was due to Jain and Kawamura [3]. As the magnetic field is increased the effective filling factor ν decreases and the quantum dot passes through a sequence of ground states. If the total angular momentum at $\nu = 1$ is L_{min}, then the filling factor at angular momentum L is given by $\nu = L_{min}/L$. The object is to predict the 'magic' values of L at which a downward cusp in the interaction energy arises, signalling a probable ground state of the quantum dot. Jain and Kawamura [3] used the ideas of composite-fermion theory together with an *ansatz* for mapping the kinetic energy of the given system to a non-interacting composite-fermion kinetic energy to determine the 'magic' L values. We used an admittedly less fundamental but extremely simple approach based on filling up of sub-Landau levels (sLLs) to predict the 'magic' L values. Profiting from the exact diagonalization results given by Kawamura *et al* in the preceding article [1], we show that our model holds to within the error limits and uncertainties indicated in our paper [2].

We briefly summarize our extremely simple method, to be called here the sLL-filling method (sLLf method), for the convenience of the reader. If the total angular momentum is L, we decompose it into $(k-1)$ units of L_{min} and the remainder R. Then we write it as, e.g.,

$$L = L_{min}(1, 1, \ldots, R/L_{min})$$

and understand it to mean that $(k-1)$ sLLs are filled, while the kth sLL has an effective filling fraction $\nu_k^* = R/L_{min}$. If R/L_{min} happens to be zero, or to correspond to one of the accepted fractional quantum Hall (FQH) filling fractions, then we say that L is a candidate for being a 'magic' L value for the quantum dot. Finite-size effects also introduce a down cusp (magic L) at $L = L_{min} + N$. This ideal sLLf scheme is modified by certain limitations and uncertainties (see [2]). For example, two quite distinct FQH fractions of the infinite

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system might give two 'magic' droplet L-values very close to each other and hence the N-particle droplet may show both, one or *neither* of these fractions. What actually happens can only be determined by a higher-order theory. For later use we call this the bad-neighbour (BN) effect. The $\nu = 1/2$ case can be approximated by many FQH fractions on both sides of $1/2$ and constitutes a pathological bad-neighbour case where the down cusp disappears as the number of electrons N increases. The sLLf method is expected to be better and BN effects become less severe as the number N of electrons in the dot becomes larger.

Our starting point for the sLL structure is indeed related to the ideas of composite fermions given by Jain [4]. However, we begin with a two-component fluid of electrons and flux quanta and construct sLLs containing electrons attached to one flux quantum each (particles) and no electrons attached to one flux quantum each (empty levels). The sLLf method given in [2] is essentially an extension of the ideas that we have successfully used in [5] to predict the ground state energies of FQH fractions of the uniform fluid. In this scheme $\nu = 1/2$ appears as an incompressible fluid with enormous fluctuations. In the droplet case with finite N these fluctuations are suppressed and $\nu = 1/2$ should appear as a down cusp, i.e. a magic L value. Thus for $N = 7$, $L_{min} = 21$, we would expect a downward cusp at $L = 42$. The simple $\Delta L = N$ rule proposed by Laughlin [6] and also by Maksym [7] would also predict a down cusp at $L = 42$. In table 1 of [2] we listed some of the down cusps for $N = 7$. The $(1, 1, 2/3)$ case is given as $L = 58$ due to an inadvertent error, and should be $L = 21+21+(21\times 2/3)$, i.e., $L = 56$, a value found in the exact diagonalization (ED) results. Some of the other possible fractions were not listed in table 1. Thus if we simply include the possible FQH and other fractions within the selected L-range discussed in [1], we have the following possible magic L-values: ($\nu = 1, L = 21$), $L = 28$, ($\nu_2^* = 5/7, L = 30$), ($\nu_2^* = 4/7, L = 33$), ($\nu_2^* = 2/3, L = 35$), ($\nu_2^* = 5/7, L = 36$), ($\nu_2^* = 6/7, L = 39$), ($\nu_2^* = 1, L = 42$), ($\nu_3^* = 1/7, L = 45$), ($\nu_3^* = 2/7, L = 48$), ($\nu_3^* = 1/3, L = 49$), ($\nu_3^* = 3/7, L = 51$), ($\nu_3^* = 4/7, L = 54$), ($\nu_3^* = 2/3, L = 56$). Of these $L = 28$ and $L = 30$ interact and $L = 30$ seems to be suppressed due to the BN effect. $L = 33$ is correctly predicted. $L = 35$ and $L = 36$ both suffer the BN effect and are not seen in the ED except as a discontinuity at $L = 36$. Then $L = 39$ is correctly predicted. $L = 42$ does not seem to appear in the ED of Kawamura *et al*. $L = 45$ and 46 suffer the BN effect and disappear. The cases $L = 45$, 51 and 56 are correctly predicted by us. Thus, indicating BN values by including them in parenthesis, we compare the prediction and exact diagonalization as follows.

Prediction :	(28, 30),	33, (35?, 36?),	39, 42,	45, (48?, 49?),	51,	(54?, 56?, 57?)
ED :	28,	33,	39,	45,	51,	56.

It is clear that our prediction scheme does a good job if the BN cusps are treated as ambiguous. In effect, a higher-order theory is needed to determine whether such cusps would survive or shift to a nearby L-value. Evidently, the composite-fermion approach of Jain *et al* [3] correctly handles most of the BN effects and provide impressive results, but *at the cost of significantly more complicated computations* than needed in our 'pocket-calculator' approach. However, as is clear from the results for the case $N = 10$, and also the $N = 6$ case, *not all* the exact diagonalization ground states are picked up by the Kawamura–Jain approach. Thus, even if we restrict ourselves to the limited L-range covered in figure 1 of [1], the $L = 61$ cusp for $N = 10$ and the $L = 40$ cusp for $N = 6$ found in the ED results [8] are not obtained by the composite-fermion mapping. This is probably because the composite fermions of Kawamura *et al* are treated as non-interacting sets of particles where N is finite. In our approach we are at a very simple level where only the existence of a gap structure is assumed and taken to be that of the infinite fluid

($N = \infty$). We had stated in our paper that the numerical estimates are probably accurate to a few per cent. This was based on the numerical data that were available to us at that time. The numerical values given for $N = 7$ and $N = 10$ by Kawamura *et al* do not seem to significantly upset that estimate. However, the results of [1] clearly establish that the cusp structure presented by our simple mapping yields more candidates for downward cusps than are actually found via a more complete theory.

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